

MATHEMATICS KANGAROO 2015

Austria - 23. 3. 2015

Level: Junior, Grades 9 – 10

Name:	
School:	
Class:	

Time: 75 min.

30 Starting points

Each correct answer to questions 1. – 10.: 3 Points

Each correct answer to questions 11. – 20.: 4 Points

Each correct answer to questions 21. – 30.: 5 Points

Each question left unanswered 0 Points

Each incorrect Answer: $\frac{1}{4}$ of the points for the question are subtracted



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Please write the letter (A, B, C, D, E) of the correct answer in the square under the question number (1 to 30). Write clearly and carefully!

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

Information über den Känguruwettbewerb: www.kaenguru.at
Wenn Du mehr in dieser Richtung machen möchtest, gibt es die Österreichische Mathematikolympiade; Infos unter: www.oemo.at

Ich melde mich zur Teilnahme zum österreichischen Wettbewerb „Känguru der Mathematik 2015“ an. Ich stimme zu, dass meine personenbezogenen Daten, nämlich Vor- und Zuname, Geschlecht, Klasse, Schulstufe, Schulstandort und Schulart 1.) zum Zweck der Organisation und Durchführung des Wettbewerbs, der Auswertung der Wettbewerbsergebnisse (Ermitteln der erreichten Punkte und Prozentzahlen), des Erstellens von schulweiten Reihungen verwendet werden.

JA NEIN

2.) zum Zweck der landes- sowie österreichweiten Reihungen, der Veröffentlichung der Ergebnisse jener Schülerinnen und Schüler, die in ihrer Kategorie zumindest 50 % der zu vergebenden Punkte erreicht haben sowie des Ermöglichens von Vergleichen mit eigenen Leistungen aus vorherigen Wettbewerbsperioden auf www.kaenguru.at verwendet werden.

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Die Zustimmung zu Punkt 2) kann nur bei einer bejahenden Zustimmung zu Punkt 1) gegeben werden. Nur Teilnehmer mit Zustimmung zu Punkt 2) werden für landes- bzw. österreichweite Siegerehrungen in Betracht gezogen.

Die Verwendung dieser Daten ist bis 31. Dezember 2017 gestattet. Diese Zustimmung kann ich gemäß § 8 Abs. 1 Z 2 DSGVO 2000 ohne Begründung jederzeit schriftlich bei webmaster@kaenguru.at widerrufen, unter Angabe folgender Informationen zur Identifizierung:

- Vor- und Zuname des Teilnehmers
- Schulstufe und Schule des Teilnehmers (genaue Adresse)
- Jahr des Wettbewerbs

Nach dem 31. Dezember 2017 werden Vor- und Zuname, die Klasse und der Schulstandort gelöscht, wobei das zuletzt genannte Datum durch die Angabe des Bundeslandes ersetzt wird. Die Verwendung der auf diese Art pseudonymisierten Daten ist nur mehr für statistische Zwecke auf der Grundlage von § 46 Abs. 1 Z 3 DSGVO 2000 erlaubt. DVR-Nummer: 300 37 06

Unterschrift:

10. A pentagon is called convex if all its internal angles are less than 180° . The number of right angles in a convex pentagon is n . Which of the following lists is a complete listing of all possible values of n ?
- (A) 1, 2, 3 (B) 0, 1, 2, 3, 4 (C) 0, 1, 2, 3 (D) 0, 1, 2 (E) 1, 2

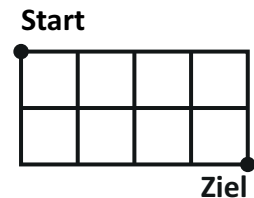
- 4 point questions -

11. In the diagram one can see my decision-die in three different positions. What is the probability I get a „YES“, when rolling this die once.



- (A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{5}{9}$ (D) $\frac{2}{3}$ (E) $\frac{5}{6}$

12. The side lengths of each of the small squares in the diagram are 1. How long is the shortest path from „Start“ to „Ziel“, if you are only allowed to move along the sides and the diagonals of the squares?



- (A) $2\sqrt{5}$ (B) $\sqrt{10} + \sqrt{2}$ (C) $2 + 2\sqrt{2}$ (D) $4\sqrt{2}$ (E) 6

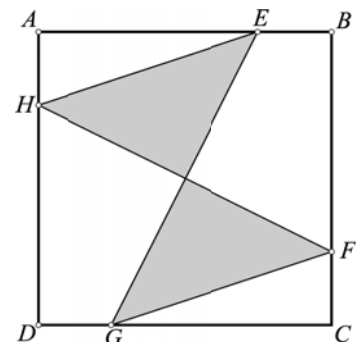
13. Each inhabitant of a distant planet has at least two ears. Three inhabitants called Imi, Dimi and Trimi meet in a trendy crater. Imi says: „I can see 8 ears.“ Dimi then replies: „I can see 7 ears.“ Finally Trimi says: „Strange, I can only see 5 ears.“ None of them can see their own ears. How many ears does Trimi have?

- (A) 2 (B) 4 (C) 5 (D) 6 (E) 7

14. A cuboid shaped container has a square base with side length 10 cm. It is filled up to a height h with water. Now a metal cube with side length 2 cm is put inside. It sinks to the bottom of the container. The water now reaches to the top corner of the metal cube. Determine h !

- (A) 1.92 cm (B) 1.93 cm (C) 1.90 cm (D) 1.91 cm (E) 1.94 cm

15. The square ABCD has area 80. The points E, F, G and H are on the sides of the square and $AE = BF = CG = DH$.



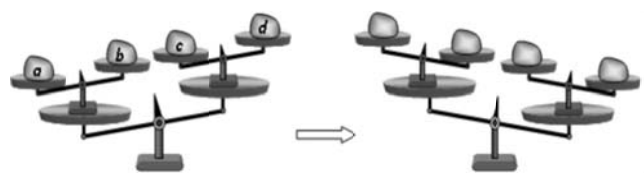
How big is the area of the grey part, if $AE = 3 \times EB$?

- (A) 20 (B) 25 (C) 30 (D) 35 (E) 40

16. If the whole number age of a father is multiplied by the whole number age of his son, one obtains 2015. Both are born in the 20th century. How big is the age gap between father and son?

- (A) 26 (B) 29 (C) 31 (D) 34 (E) 36

17. Four objects a, b, c, d are placed on a double balance as shown. Then two of the objects are exchanged, which results in the change of position of the balance as shown. Which two objects were exchanged?



- (A) a and b (B) b and d (C) b and c (D) a and d (E) a and c

18. It is known that the solutions of the quadratic equation $x^2 - 85x + c = 0$ are prime numbers. What is the digit sum of c ?

- (A) 12 (B) 13 (C) 14 (D) 15 (E) 21

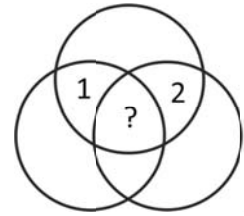
19. How many three-digit positive whole numbers are there, where the digits when placed side by side always differ by 3?

- (A) 12 (B) 14 (C) 16 (D) 20 (E) 27

20. Which value of the variable n is a counterexample to the statement „If n is a prime number, then exactly one of the two numbers $n - 2$ and $n + 2$ is a prime number.“?
- (A) 11 (B) 19 (C) 21 (D) 29 (E) 37

- 5 point questions -

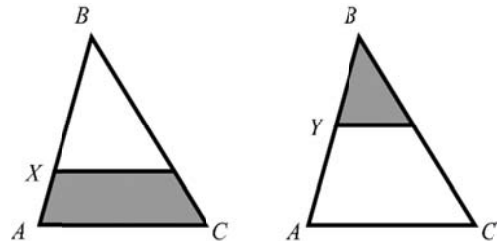
21. In the diagram we can see seven sections which are bordered by three circles. One number is written into each section. It is known that each number is equal to the sum of all the numbers in the adjacent zones. (Two zones are called adjacent if they have more than one corner point in common.) Which number is written into the inner area?
- (A) 0 (B) -3 (C) 3 (D) -6 (E) 6



22. How many two-digit numbers can be written as sum of exactly six different powers of two? (Hint: Powers of two are $2^0, 2^1, 2^2, \dots$)
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

23. Petra has three different dictionaries and two different novels on her bookshelf. In how many different ways can she arrange the books, if all the dictionaries should stay together and likewise the novels as well?
- (A) 12 (B) 24 (C) 30 (D) 60 (E) 120

24. Lines parallel to the base AC of triangle ABC are drawn through X and Y . In each case, the areas of the grey parts are equal in size. The ratio $BX : XA = 4 : 1$ is known. What is the ratio $BY : YA$?
- (A) 1 : 1 (B) 2 : 1 (C) 3 : 1 (D) 3 : 2 (E) 4 : 3



25. In a right-angled triangle the angle bisector of an acute angle splits the opposite side into segments of length 1 and 2 respectively. How long is this angle bisector?
- (A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt{4}$ (D) $\sqrt{5}$ (E) $\sqrt{6}$

26. A two-digit number with the digits x, y , can be written in the form \overline{xy} . Let a, b, c be different digits. In how many way can the digits a, b, c be chosen, so that $\overline{ab} < \overline{bc} < \overline{ca}$?
- (A) 84 (B) 96 (C) 125 (D) 201 (E) 502

27. If one of the numbers $1, 2, 3, \dots, n - 1, n$, is crossed out, the average of the remaining numbers is 4.75 . Which number was crossed out?
- (A) 5 (B) 7 (C) 8 (D) 9 (E) The number cannot be determined for certain.

28. The ant Tanti starts an adventure at a vertex of a cube with side length 1. She wants to walk along each edge of the cube at least once and return to the starting point at the end. What is the minimum possible length of her walk?
- (A) 12 (B) 14 (C) 15 (D) 16 (E) 20

29. Ten different numbers are written down. Each number which is equal to the product of the other nine numbers can then be underlined. What is the maximum amount of numbers that can be underlined?
- (A) 1 (B) 2 (C) 3 (D) 9 (E) 10

30. Several points are marked on a straight line. Then all possible connecting lines between each two points are drawn. One such point lies within exactly 80 of those connecting lines, and another one lies within exactly 90 of those. How many points were marked on the straight line?
- (A) 20 (B) 22 (C) 80 (D) 90 (E) Not enough information given to determine an answer.