

# Känguru der Mathematik 2010

## Group Student (From Grade 11.)

### Austria - 18.3.2010



#### - 3 Point Questions -

1) In the picture opposite we see that  $1+3+5+7 = 4 \times 4$ . How big is  $1+3+5+7+\dots+17+19$ ?

- A)  $10 \times 10$       B)  $11 \times 11$       C)  $12 \times 12$       D)  $13 \times 13$       E)  $14 \times 14$

2) Which number goes in the cell with the question mark if the sum of the numbers in both rows is equal?

1	2	3	4	5	6	7	8	9	10	2010
11	12	13	14	15	16	17	18	19	20	?

- A) 1010      B) 1020      C) 1910      D) 1990      E) 2000

3) The hollow spaces of two empty containers are cubic and have a base area of  $1 \text{ dm}^2$  and  $4 \text{ dm}^2$  respectively. The big container is to be filled with water, using the small one as a scoop. How many full scoops are necessary to fill the big cube?

- A) 2      B) 4      C) 6      D) 8      E) 16

4) How many four-digit numbers, made up of odd digits only, are divisible by 5?

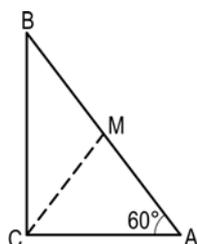
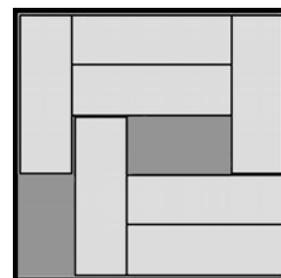
- A) 900      B) 625      C) 250      D) 125      E) 100

5) The managing director of a company claims „Every one of our employees is at least 25 years old.“ It turns out, he is wrong. Which of the following statements is correct?

- A) All employees of the company are exactly 25 years old.  
 B) All employees of the company are more than 26 years old.  
 C) No employee of the company is already 25 years old.  
 D) One of the employees of the company is less than 25 years old.  
 E) One of the employees of the company is exactly 26 years old.

6) In the box are seven blocks. You want to rearrange the blocks so that another block can be placed. What is the minimum number of blocks that have to be moved?

- A) 2    B) 3    C) 4    D) 5    E) It is not possible.



7) The triangle pictured is right-angled. M is the midpoint of the hypotenuse AB and  $\angle BCA = 90^\circ$ . How big is  $\angle BMC$ ?

- A)  $105^\circ$       B)  $108^\circ$       C)  $110^\circ$       D)  $120^\circ$       E)  $125^\circ$

8) Which of the following numbers could be the number of edges of a prism?

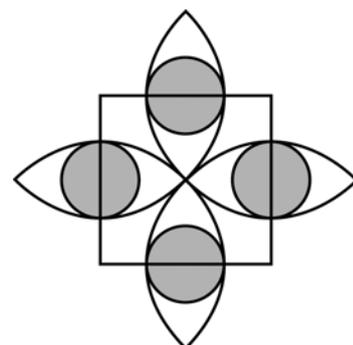
- A) 100      B) 200      C) 2008      D) 2009      E) 2010

9) How many two-digit numbers with x in the tens-column and y in the unit-column have the properties  $(x-3)^2 + (y-2)^2 = 0$ ?

- A) 1      B) 2      C) 6      D) 32      E) none

10) In the figure the square has side length 2. The semi-circles pass through the midpoint of the square and have their centres on the corners of the square. The grey circles have their centres on the sides of the square and touch the semi-circles. How big is the total area of the grey parts?

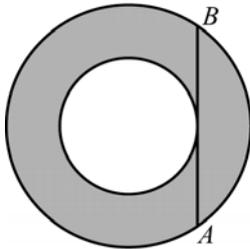
- A)  $4 \cdot (3 - 2\sqrt{2}) \cdot \pi$     B)  $\sqrt{2} \cdot \pi$     C)  $\frac{\sqrt{3}}{4} \cdot \pi$     D)  $\pi$     E)  $\frac{1}{4} \cdot \pi$



**- 4 Point Questions -**

11) The numbers  $\sqrt{7}$ ,  $\sqrt[3]{7}$  und  $\sqrt[5]{7}$  are, in this order consecutive terms of a geometric sequence. Determine the next term.

- A)  $\sqrt[7]{7}$       B)  $\sqrt[12]{7}$       C)  $\sqrt[5]{7}$       D)  $\sqrt[10]{7}$       E) 1



12) The chord AB touches the smaller of the two concentric circles. The length AB = 16. How big is the area of the grey part?

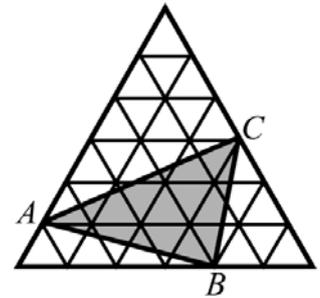
- A)  $32\pi$     B)  $63\pi$     C)  $64\pi$     D)  $32\pi^2$     E) It depends on the radius of the circles.

13) The integers x and y fulfill the condition  $2x = 5y$ . Only one of the following numbers can be considered for x+y. Which?

- A) 2011    B) 2010    C) 2009    D) 2008    E) 2007

14) The big equilateral triangle consists of 36 small equilateral triangles which each have an area of 1 cm<sup>2</sup>. Determine the area of ABC.

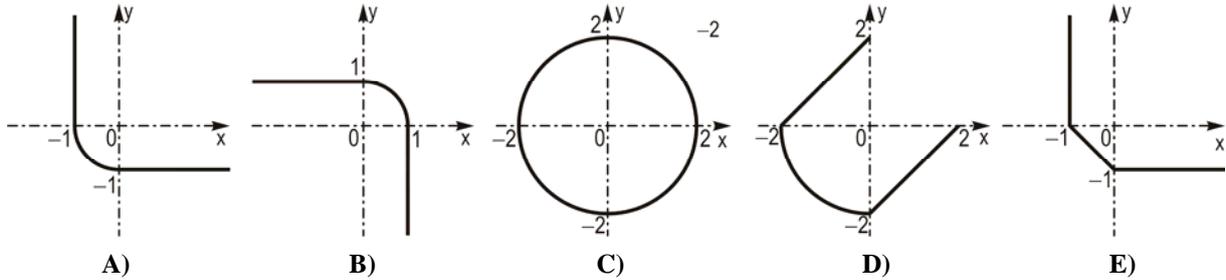
- A) 11 cm<sup>2</sup>    B) 12 cm<sup>2</sup>    C) 13 cm<sup>2</sup>    D) 14 cm<sup>2</sup>    E) 15 cm<sup>2</sup>



15) In a bag are blue, green and red balls (at least one ball of each colour). If we randomly take five balls out of the bag, we know: At least two balls are red and at least three are of the same colour. How many blue balls are in the bag?

- A) 1    B) 2    C) 3    D) 4    E) Not possible to answer.

16) Which of the following graphs represents the solution set of  $(x-|x|)^2 + (y-|y|)^2 = 4$ ?



17) If we connect three cornerpoints of a regular 14-sided polygon then a triangle is created. How many of those triangles are right-angled?

- A) 42    B) 84    C) 88    D) 98    E) 168

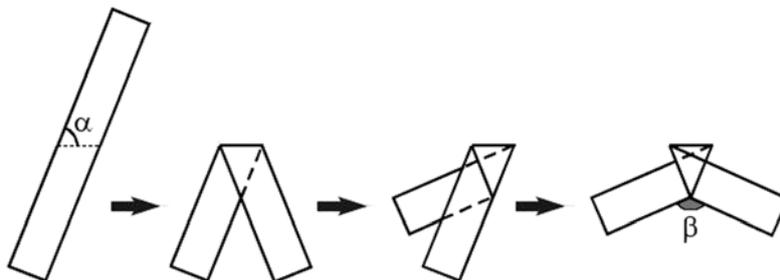
18) Each star in the expression  $1*2*3*4*5*6*7*8*9*10$  is either replaced by a „+“ or a „x“. Let N be the biggest number possible that can be obtained this way. What is the smallest prime factor of N?

- A) 2    B) 3    C) 5    D) 7    E) Another number

19) The side-lengths of a triangle in cm are given by the natural numbers 13, x and y. Determine the perimeter of the triangle if  $xy = 105$ .

- A) 35    B) 39    C) 51    D) 69    E) 119

20) A strip of paper is folded three times as shown. Determine  $\beta$  if  $\alpha = 70^\circ$ .

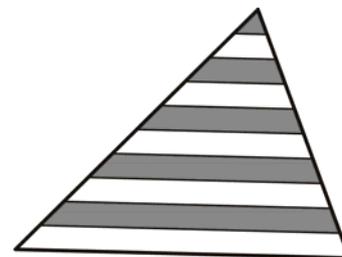


- A) 140°    B) 130°    C) 120°    D) 110°    E) 100°

**- 5 Point Questions -**

21) Lines drawn parallel to the base of the triangle pictured separate the two other sides into 10 sized parts. What percentage of the triangle is grey?

- A) 42.5 %      B) 45 %      C) 46 %      D) 47.5 %      E) 50 %



22) 100 people take part in a race where no-one can tie. Everybody is questioned after the race as to which place they have achieved and all answer with a number between 1 and 100. The sum of all answers is 4000. What is the minimum number of people who have lied about their result?

- A) 9      B) 10      C) 11      D) 12      E) 13

23) I roll an ordinary die once. What is the probability that I rolled '2' at least once under the condition that the third number is equal to the sum of the first two?

- A)  $\frac{1}{6}$       B)  $\frac{91}{216}$       C)  $\frac{1}{2}$       D)  $\frac{8}{15}$       E)  $\frac{7}{12}$

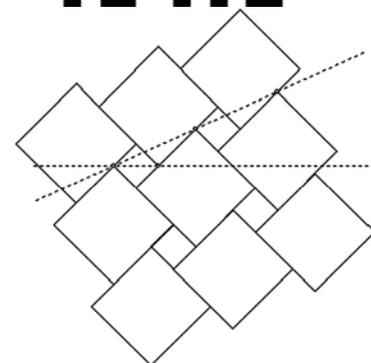
24) A barcode as pictured is made up of alternate black and white stripes. The code always starts and ends with a black stripe. Each strip (black or white) has the width 1 or 2 and the total width of the barcode is 12. How many different barcodes of this kind are there if one reads from left to right?

- A) 24      B) 132      C) 66      D) 12      E) 116



25) The picture on the right shows a tile pattern. The side length of the bigger tiles is a and of the smaller ones b. The dotted lines (horizontal and tilted) include an angle of 30°. How big is the ratio a:b?

- A)  $(2 \cdot \sqrt{3}):1$       B)  $(2 + \sqrt{3}):1$       C)  $(3 + \sqrt{2}):1$   
 D)  $(3 \cdot \sqrt{2}):1$       E) 2:1



26) The numbers from 1 to 10 are written 10 times each on a board. Now the children play the following game: One child deletes two numbers off the board and writes instead the sum of the two numbers minus 1. Then a second child does the same, and so forth until there is only one number left on the board. The last number is

- A) less than 440.      B) 451.      C) 460.      D) 488.      E) greater than 500.

27) The expression  $\frac{(2+3)(2^2+3^2)\dots(2^{1024}+3^{1024})(2^{2048}+3^{2048})+2^{4096}}{3^{2048}}$  is equal to

- A)  $2^{2048}$       B)  $2^{4096}$       C)  $3^{2048}$       D)  $3^{4096}$       E)  $3^{2048} + 2^{2048}$

28)  $\sqrt[100]{0.44\dots4}$  is written as a decimal. What is the 100th digit after the decimal point?

- A) 1      B) 2      C) 3      D) 4      E) 6

29) A function maps all positive real numbers to real numbers. For all x the following holds true:  $x \in \mathbb{R}^+$  :

$$2f(x) + 3f\left(\frac{2010}{x}\right) = 5x . \text{ Determine the value of } f(6).$$

- A) 993      B) 1      C) 2009      D) 1013      E) 923

30) On the two catheti of a right-angled triangle (with lengths a and b respectively) points P and Q respectively are chosen. Let K and H be the endpoints of the perpendicular lines from P and Q respectively, to the hypotenuse of the triangle. How big is the smallest possible value of KP + PQ + QH?

- A) a+b      B)  $\frac{2ab}{a+b}$       C)  $\frac{2ab}{\sqrt{a^2+b^2}}$       D)  $\frac{(a+b)^2}{\sqrt{a^2+b^2}}$       E)  $\frac{(a+b)^2}{2ab}$

# KÄNGURU DER MATHEMATIK 2010

## 18.3.2010

Categorie: Student, Grades: 11-13

Name:	
School:	
Class:	

Time allowed: 75 min.

Each correct answer questions 1.-10.: 3 Points

Each correct answer questions 11.-20.: 4 Points

Each correct answer questions 21.-30.: 5 Points

Each question with no answer given: 0 Points

Each incorrect answer: Lose  $\frac{1}{4}$  of the points for that question

Therefore 30 starting points



**Please write the letter (A, B, C, D, E) of the correct answer under the question number (1 to 30)**

**Write neatly and clearly!**

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

Information über den Känguruwettbewerb: [www.kaenguru.at](http://www.kaenguru.at)

Wenn Du mehr in dieser Richtung machen möchtest, gibt es die Österreichische Mathematikolympiade; Infos unter:

[www.oemo.at](http://www.oemo.at)